**Algorithms - Homework 1**

**You need to submit a printout of your code (in one menu based) before the next class (sadly we can’t take any submission by now, as LMS registration is under process). Keep your code by yourself.**

Given an N-1- element array, Finding an element from within a range of 1-N.

**Question 1**

1. **(Code Submission required)** N^2 Algorithm - Finding each value from within the range into the array. **(10 points)**
2. **(Code Submission required)** N log N + N log N ( Sorting + binary Searching). **(10 points)**
3. **(Code Submission required)** N log N + N (Sorting + One Scan Algorithm). **(10 points)**
4. **(Code Submission required)** **N (creation of boolean array filled with false values)** + N(Boolean array filling, with respect to each value) + N(Searching a value which one is missing by scanning Boolean array). **(10 points)**
5. **(Code Submission required)** N (reading) +N( Summing from 1 to N) + N(Subtracting each value from Sum variable). **(10 points)**
6. **(Pseudocode Required) Bit Complexity and analysis of the above 5 solutions** 
   1. 2N Bit reading Algorithm finding the missing element. **(25 points)**

**Question 2**

1. **Write the code for binary search, both recursive and iterative**
2. **Write the Growable Array and Vector Implementation compare the time comparison between both by taking 10MB and load it.**
3. **Now use load of Vector to read the data.**

**THE CHALLENGE (This is due by weekend) (25 points)**

You wake up after your Titanic sank, and you find yourself on the shore of the sea, an island. Beside everything, what an island has (its natural beauty), there are tribesmen there too. You come to know that that every tribesman of this island is either faker(he may or may not lie) or trustworthy(always say the truth). You can ask them questions like who is a faker. And who is trustworthy? There are total of 100 people in this island. And you can ask as many questions as you want. The only information known is, that number of Trustworthiness is greater than fakers. Any answer having greater than 500 questions is not worthy enough to be considered. Figure out who is a faker and who is a trustworthy person?

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| **The Geometric Series**  Geometric Series is one of the most essential and exists in abundance in nature and also in computational related problems in CS. It looks like the following:  **1+ b + b2 + b3 + b4 + … + N/b2 + N/b1 + N** or also written as **N + N/b + N/b2 + … + b4+b3+b2+b+1**  Left side is known as increasing geometric series and right side as decreasing geometric series both ending and starting with N.  It is not hard to prove that that the summation of the above series is bounded by a constant time the largest term i.e. some C x N. See Wikipedia for the proof [**https://en.wikipedia.org/wiki/Geometric\_series**](https://en.wikipedia.org/wiki/Geometric_series)  Example of the above series could be the following series with b = 2. **1+2+4+8+16+...+ N/4 + N/2 + N** also written as **N + N/2 + N/4 + N/8 + ….+ 16 + 8 + 4 + 2 + 1**  It is not hard to prove that the summation of the above series (2 version increasing as well as the decreasing geometric series is always less than 2N. The proof is quite simple and very simple to figure out what it means. Imagine that the largest term N is actually one big Square S take two big squares now the claim is the rest of the summation can never together sum up and fill in the 2nd square. Very simply start shading the summed up region. If N is a complete square S then N/2 will be half of the square. Similarly N/4 will be the quarter of the square S and so on. Hence you will never be able to fill in the 2nd square S. That completes the proof that the summation of the geometric series with increasing or decreasing factor b=2 is always bounded by <2N. |

**Directions to look at of THE CHALLENGE**

You have to find at least one trustworthy by whom you can ask who is a trustworthy or lier. The strategy is inspired from the beautiful Geometric series. One idea could have been to take one person out and ask from everyone “Is he the trustworthy one?” if majority say yes then we have found the one trustworthy and we may ask him about everyone. But the issue is you could be unlucky in that case you have to choose the 2nd person and repeat that would be a very bad solution. It will work but in that case you may have to almost select 49 bad choices of fakers.   
Now the idea is make pair of 2 each (resulting 50 pairs i.e. N/2 pairs) and just ask those pairs about each other Here could be the possible answers (F for faker, T for Trustworthy).

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| Case 1 | F | F |
| Case 2 | T | F |
| Case 3 | T | T |

In all those cases where cases 1 and 2 arise we reject the pairs. In case 3 we choose one representative and ask the other person in the pair to sit down. With this one process greater than half of the people have been pruned and you should prove that “the Trustworthy-men in majority” condition will still be valid (you should prove that?). With this one step we have asked 100 questions but the searching space have reduced to half, now we repeat the same process of shrinking and on the worst case the number of questions will look like the following series

100 + 50 + 25 + … + 1 that will always be less than 200 questions. Hence we have found one Trustworthy in around 200 questions.

Be careful that with this procedure it may be possible that there are odd number of people remaining on a certain step what should we do in that case? We have to think about that. Hint is in incorporating that issue we might have to do some extra steps which might take the number of questions close to 300.